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The Mystery and Clarity of Leonardo da Vinci's Coefficient of Friction

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A B S T R A C T

The science of friction has been using the coefficient of friction as the main quantitative characteristic of the friction process for more than five centuries. The concept of the coefficient of friction as a characteristic of the resistance to the movement of rubbing surfaces in a hidden form was formulated by Leonardo da Vinci: "Each rubbing body exerts a friction resistance equal to one quarter of its weight, provided that the flat plane is in contact with the polished surface". Two centuries later, the coefficient of friction appeared explicitly, in the form of a formula, in the works of G. Amontons. It is clear that the coefficient of friction is a convenient friction parameter, easily determined in the experiment. However, what is the physical meaning of resistance to the movement of rubbing surfaces? Modern engineering believes that the coefficient of friction has no physical meaning. Thermodynamic analysis of the friction process is performed. The evolutionary patterns of the behavior of the friction contact are shown. A structural-energy interpretation of the logic of the coefficient of friction (resistance) is proposed: accumulation of potential energy of defects in the crystal structure of the deformable volume by friction contact. The static potential energy of the formed defects in the crystal structure of the contact is a measure of the decrease in the kinetic energy of the relative motion of the rubbing surfaces. Such a formulation of resistance to movement under friction has a deep physical and clear meaning for a scientist and engineer. The substantiation of Leonardo da Vinci's formulation of friction that the friction resistance should be equal to 0.25 is given. The interpretation of this rule is given for the case of friction in mechanisms (machines).

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1. INTRODUCTION

Leonardo da Vinci as the author of the concept of the coefficient of friction is both clear and mysterious.

Indeed, the coefficient of friction as the ratio of the friction force to the normal load pressing the friction surface is a convenient way for an engineer to assess the resistance to the relative movement of these surfaces. Leonardo da Vinci's

recommendations that bodies have different sliding abilities are clear here.

Another recommendation of Leonardo da Vinci that "Every rubbing body exerts a friction resistance equal to one quarter of its weight, provided that a flat plane comes into contact with a polished surface" [1] is perceived mysteriously today.

It turns out that the coefficient of friction must simultaneously be different for the friction pair and also be a certain constant for all friction pairs?! Leonardo da Vinci associates the concept of this friction constant with a certain condition of the friction process - the contact of a flat plane with a polished surface…

Naturally, there is considerable scientific interest in understanding the logic of Leonardo da Vinci's recommendation regarding the concept of the coefficient of friction as a constant equal to 0.25. Indeed, what is the physical meaning of the coefficient of friction, which makes it possible to understand its clarity and mystery?

In the most general sense, the expression for the coefficient of friction from Leonardo da Vinci as a resistance parameter (two centuries later presented by G. Amontons in the formula familiar to us) does not carry any explicit physical meaning.

On the other hand, is it possible to consider any physical meaning in Leonardo da Vinci's coefficient of friction?

Is it possible to link the logic of the coefficient of friction to the concept of friction as a global phenomenon of nature, i.e. the phenomenon of transformation and energy dissipation of external relative motion of friction surfaces?

What is the essence of resistance to the movement of surfaces?

Here the question arises. Can we, in the power interpretation of the coefficient of friction, understand its physical meaning precisely as a parameter of resistance to the movement of surfaces (friction)? If this is not possible, then in which physical model should the expression for the coefficient of friction be presented? Here we see the following. Friction is a global phenomenon of transformation and energy dissipation of rubbing surfaces. Therefore, it is advisable to analyze the friction process from the standpoint of the global law of nature – the law of conservation of energy during its transformations.

Let's consider the possibility of a structural-energy interpretation of the coefficient of friction from the standpoint of the equation of the energy balance of friction.

Friction will be considered as a process of joint elastic-plastic deformation and destruction of contact volumes of rubbing surfaces, complicated by the influence of the environment. A global thermodynamic method can be used as a method adequate to the globality of the friction phenomenon, namely, the equations of the energy balance of the process at the friction contact.

2. THERMODYNAMIC ANALYSIS OF THE FRICTION PROCESS

2.1 Structural and energetic interpretation of the process of plastic deformation and destruction of solids

We know the theoretical and experimental scheme (Fig. 1) of the energy balance of the process of plastic deformation and destruction of solids, obtained by the scientific school of Professor V.V. Fedorov [2,3] based on the results of the study of cyclic (fatigue) volumetric tensile strength and abrasive wear (grinding) of solids.

From a thermodynamic point of view, the whole variety of mechanisms and structural levels of plastic flow can be divided into two most characteristic groups - adaptive and dissipative types.

The first group should include the mechanisms of nucleation and accumulation in local volumes of metastable structures (atom-vacancy pairs and other defects). The integral characteristic of the intensity of these processes is the specific (attributed to a unit of volume) pumping power $\dot{u}_e$ of excess (latent) energy.

The second group includes elementary acts of atomic-molecular rearrangements that control dynamic return. The integral characteristic of the intensity of these processes is the specific power of dissipative processes (thermal effect) $\dot{q}$ controlling the quasi-viscous flow of the body element

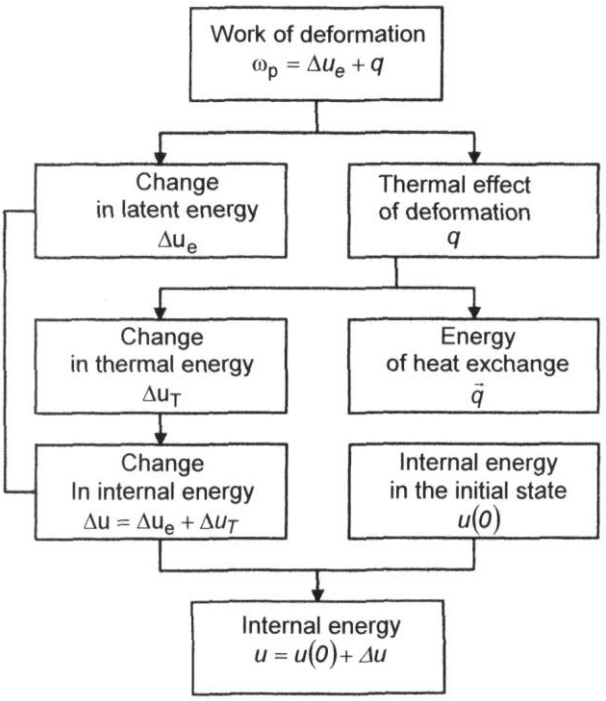


Fig. 1. Diagram of the energy balance of the process of plastic deformation and destruction of solids [2,3].

A thermodynamic condition of local destruction is formulated and an integral criterion of damage is proposed [2]. As a parameter of damage (scattered destruction), the density of the internal energy u accumulated in deformable volumes is taken, determined by the sum of two terms: potential (latent) u_e and kinetic (thermal) u_T energies.

$$u = u_e + u_T . \quad (1)$$

This energy is associated with the accumulation of static Δu_e and dynamic Δu_T damages and distortions of the crystal lattice in the deformable element of the body, therefore, it is responsible for scattered destruction (damage).

An element of a body is considered destroyed if, in at least one local micro-volume responsible for the destruction, the internal energy density reaches a critical (limit) value u_* corresponding to the loss of stability by the crystal lattice "in large" [4, 5]. This moment corresponds to the appearance of a crack of critical size in the local micro-volume (according to Griffiths-Orovan-Irwin) and a sharp localization of the process at the mouth (top) of the crack.

The thermodynamic condition of local destruction is written as

$$u(\bar{r}_*, t_*) = u(\bar{r}_*, 0) + \int_0^{t_*} \dot{u}\ (\bar{r}_*, t)dt = u_* = const , \quad (2)$$

where $u(\bar{r}_*, 0)$ is the density of internal energy in the local volume in the original (before deformation $t = 0$) state; $\dot{u}(\bar{r}_*, t)$ - the specific rate of change of internal energy in the local volume responsible for the destruction; $\bar{r}_*$ - a parameter characterizing the coordinates (x_*, y_*, z_*) of the local volume responsible for the destruction.

The thermodynamic criterion of limiting (highly excited) states is substantiated. From a thermodynamic point of view, there is an energy analogy between the processes of mechanical destruction and melting of metals and alloys [6]. In accordance with this, the internal energy u_* accumulated in deformable macro volumes of the material responsible for mechanical destruction should be considered as the sum of two terms:

1) the specific energy $u_*^{'}$ of static distortions equivalent to dynamic distortions in the deformable volume V of the crystal when it is heated to the melting point T_S, i.e.

$$u'_* = \Delta H_{solid} = \int_0^{T_S} \rho\ c_p dT , \quad (3)$$

where ρ and c_p is the density and heat capacity of the material; ΔH_{solid} - enthalpy of the material in the solid state at the melting point T_S;

2) the specific energy u''_* consumed for the crystal-amorphous phase transition in the local volume V_* of the material responsible for destruction at a melting temperature T_S equivalent to the latent heat of melting L_S, i.e.

$$u''_* = \frac{V_*}{V} L_S . \quad (4)$$

According to this

$$u_* = u'_* + u''_* = \int_0^{T_S} \rho\ c_p dT + \frac{V_*}{V} L_S . \quad (5)$$

Depending on the structure of the material and the conditions of deformation, the ratio V_*/V can vary widely $0 \le V_*/V \le 1{,}0$, therefore, the critical value of the internal energy density u_* absorbed by the deformable volume V at the time of destruction (formation of a crack of critical size) can vary within

$$\Delta H_{solid} \le u_* \le \Delta H_S . \qquad (6)$$

In the local volume responsible for the destruction $V_*/V \to 1{,}0$, therefore $u_* = \Delta H_S$.

Here - enthalpy in solid ΔH_{solid} and liquid ΔH_S states.

The Table1 below shows the values of the internal energy densities corresponding to the limiting state of solid deformable bodies.

Table 1. Results of statistical processing of experimental data on the critical internal energy density during surface destruction (dispersion) of metals [2].

Characteristics: material, heat treatment mode	Average sample value $\Delta\bar{u}_* \cdot 10^{-2}, MJ/m^3$	Mean square deviation $S_u \cdot 10^{-2}, MJ/m^{-3}$	Coefficient of variation γ, %
40X steel, annealing	96,14	13,1	13,6
У8А steel, annealing	108,20	24,5	22,7
Steel У8А, quenching	74,60	8,2	11,0
Cast iron СЧ18-36, annealing	88,17	27,6	31,3
Brass Л68, annealing	44,0	7,0	15,0
Aluminum alloy А09-2, annealing	11,60	2,3	19,0
БрОЦС 3-12-5, annealing	27,05	5,0	18,3
The designation of grades of alloys is Russian			

Iron (Fe_α): $\Delta H_S = \Delta H_{Solid} + L_S = 88{,}8 \cdot 10^2 + 21{,}2 \cdot 10^2 = 105 \cdot 10^2 \ \ MJ/m^3$

2.2 The relationship between the stress-strain and energy state of the body element

An unambiguous relationship has been established [2] between the stress-strain and energy state of the body element:

$$\dot{\varepsilon}_p = \frac{\dot{\omega}_p}{\sigma_i} = \frac{1}{\sigma_i}\left(\dot{u}_e + \dot{q}\right) = \dot{\varepsilon}_e + \dot{\varepsilon}_q , \qquad (7)$$

$$\varepsilon_p = \frac{\omega_p}{\sigma_i} = \frac{1}{\sigma_i}\left(\Delta u_e + q\right) = \varepsilon_e + \varepsilon_q . \qquad (8)$$

Here $\varepsilon_p, \dot{\varepsilon}_p$ - the plastic component of the complete deformation of the body and the rate of plastic deformation.

Therefore, from a thermodynamic point of view, the total amount of accumulated plastic deformation ε_p and the rate of its change $\dot{\varepsilon}_q$ can be represented as the sum of two terms associated, respectively, with deformation hardening and damageability $(\varepsilon_e,\ \dot{\varepsilon}_e)$ and dynamic return $(\varepsilon_q,\ \dot{\varepsilon}_q)$, controlling the quasi-viscous flow of the material.

This important conclusion is of fundamental importance when analyzing the relationship between the processes controlling plastic deformation and destruction of a solid and when choosing criteria for local destruction. Only a part of the plastic (irreversible) deformation $\varepsilon_e = \varepsilon_p - \varepsilon_q$, controlled by microscopic processes that are associated with deformation hardening and accumulation of latent energy of defects and damages, is responsible for the damageability and destruction of the body element.

A significant part of the irreversible deformation $(\varepsilon_q = \varepsilon_p - \varepsilon_e)$, controlled by relaxation (dissipative) processes, does not affect the damageability and destruction of the body element, but only causes its quasi-viscous flow (stationary creep, superplasticity - at stresses less than critical $\sigma_i < S_{0*} = \left(\nu\ u_e/\alpha\right)^{0{,}5}$). Here σ_i - intensity of stresses; S_{0*} - real solid strength;

ν, α - structural parameters characterizing the locality and heterogeneity of elementary acts in terms of the volume of the body element [2].

The relationship between the work and the degree of irreversible deformation and their components varies very widely and depends on the structure of the material and the conditions of its deformation [2].

2.3 Equations of the energy balance of the friction process

According to the scheme of the energy balance of the process of plastic deformation and destruction (Fig.1), the work w_f of external forces F during friction on the path l for a unit of deformable (contact) volume [7] can be represented as follows:

$$w_f = w_{plast} = \Delta u_e + q$$

or

$$\dot{w}_f = \dot{w}_{plast} = \dot{u}_e + \dot{q}. \qquad (9)$$

Here $\dot{w}_f = dw_f / dt$ is the power of friction (dissipation) of energy; $\dot{u}_e = du_e / dt$ - the rate of accumulation of latent energy in deformable (contact) volumes; $\dot{q} = dq / dt$ - the power of the thermal effect of plastic deformation (friction).

Let's select a common small point on the contact of two irregularities (Fig.2) and write down the following for the contact volume of this point belonging to both materials that make up the friction pair:

$$w_f = \Delta u_{e1} + \Delta u_{e2} + q_1 + q_2; \qquad (10)$$

$$\dot{w}_f = \dot{u}_{e1} + \dot{u}_{e2} + \dot{q}_1 + \dot{q}_2. \qquad (11)$$

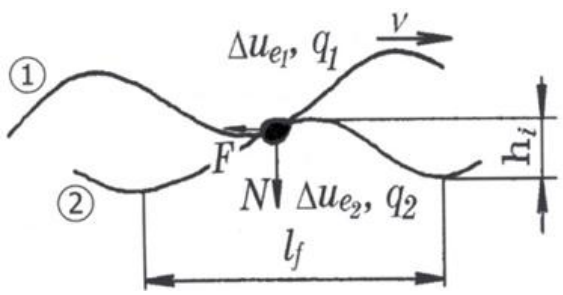


Fig. 2. Conditional scheme of contact of solids under friction [7].

The obtained equations (7) - (9) represent the friction process as a complex contradictory (competitive) dialectical phenomenon characterized by the simultaneous occurrence in deformable (contact) volumes of materials of a friction pair of two interrelated and opposite groups of microscopic processes.

These equations (9)-(11) show that from the thermodynamic point of view, the work of the friction forces w_T (friction power $\dot{w}_f$) it can be divided into two specific parts. The first part of the friction work w_f is associated with a change in the deformable (contact) volumes of materials of the latent (potential) energy Δu_{e1}; Δu_{e2} of various kinds of elementary defects and damages arising and accumulating in deformable volumes. This energy is an unambiguous and integral characteristic of submicro-and microstructural changes that occur in plastically deformable volumes of materials [2,3]. It is a measure of deformation hardening and damageability of materials. The second part of the friction work w_f is related to the processes of dynamic return, accompanied by the release of latent energy and the thermal effect q_1, q_2 of friction. This energy is associated with the movement and destruction of various kinds of elementary defects of opposite signs, their appearance on the surface, the healing of reversible submicroscopic violations of continuity, etc.

According to [2,7] and Fig. 1, the thermal effect of friction should be represented as the sum of two components Δu_T and $\vec{q}$. The first component Δu_T represents the thermal effect of friction in the form of the effect of self-heating of the contact volume with temperature T_f:

$$\Delta u_T = \int_{\theta}^{T_f} \rho\, c\, dT, \qquad (12)$$

where θ is the temperature of the material before deformation; ρ, c - density and heat capacity of the material.

The second component $\vec{q}$ represents the so-called "transit" component [2] of the thermal friction effect, which essentially characterizes the dynamic dissipative component of friction [7].

Thus, according to [2], the thermal effect of plastic deformation under friction should be represented as the sum of two components: the component of static and dynamic dissipation

$$q = \Delta u_T + \vec{q}\,. \tag{13}$$

The internal energy Δu accumulated in the deformable volume of the body is also determined by the sum of two components: potential (latent) Δu_e and kinetic (thermal) Δu_T:

$$\Delta u = \Delta u_e + \Delta u_T\,. \tag{14}$$

This energy [2] is associated with the damageability Δu_e of the material and its thermal softening Δu_T, i.e. it is dangerous and responsible for destruction.

These friction components should be identified as a component of mechanical activation and damageability and a component of thermal activation and damageability. All intermediate values in the ratio between the potential Δu_e and thermal Δu_T components of the change Δu in damageability (destruction) relate to either quasi-brittle or quasi-viscous destruction (damageability).

As a result, the relations for the work of friction and the friction power (9)-(11) should be presented in the most general form

$$w_f = \Delta u_{e1} + \Delta u_{e2} + \Delta u_{T1} + \Delta u_{T2} + \vec{q}_1 + \vec{q}_2\,; \tag{15}$$

$$\dot{w}_f = \dot{u}_{e1} + \dot{u}_{e2} + \dot{u}_{T1} + \dot{u}_{T2} + \dot{\vec{q}}_1 + \dot{\vec{q}}_2\,. \tag{16}$$

In the most general case, equations (10) and (11) should be presented taking into account the real (not a single) dimensions of the tribocontact V_{f1} and V_{f2}:

$$W_f = \Delta U_{e1} + \Delta U_{e2} + Q_1 + Q_2 = \\ = \Delta U_{e\Sigma} + Q_\Sigma\,; \tag{17}$$

$$\dot{W}_f = \dot{U}_{e1} + \dot{U}_{e2} + \dot{Q}_1 + \dot{Q}_2 = \dot{U}_{e\Sigma} + \dot{Q}_\Sigma\,, \tag{18}$$

where $\Delta U_{e\Sigma} = \Delta u_{e1} V_{f1} + \Delta u_{e2} V_{f2}$ and $\dot{U}_{e\Sigma} = \dot{u}_{e1} V_{f1} + \dot{u}_{e2} V_{f2}$ is the change and rate of change of the accumulated latent energy in the contact volumes of the friction pair; Q_Σ and $\dot{Q}_\Sigma$ is the thermal effect of friction and its power; V_{f1}, V_{f2} - contact (deformable) volumes of materials of the friction pair (Fig. 2).

Solving equations (17) and (18) with respect to the friction force, we obtain generalized equations for the friction force:

$$F_l = \frac{\Delta U_{e1} + \Delta U_{e2} + Q_1 + Q_2}{l} = \\ = \frac{\Delta U_{e\Sigma}}{l} + \frac{Q_\Sigma}{l}\,; \tag{19}$$

$$F_t = \frac{\dot{U}_{e1} + \dot{U}_{e2} + \dot{Q}_1 + \dot{Q}_2}{v} = \\ = \frac{\dot{U}_{e\Sigma}}{v} + \frac{\dot{Q}_\Sigma}{v}\,. \tag{20}$$

Accordingly, we present generalized equations for the coefficient of friction μ:

$$\mu_l = \frac{\Delta U_{e1} + \Delta U_{e2} + Q_1 + Q_2}{Nl} = \\ = \frac{\Delta U_{e\Sigma}}{Nl} + \frac{Q_\Sigma}{Nl}\,; \tag{21}$$

$$\mu_t = \frac{\dot{U}_{e1} + \dot{U}_{e2} + \dot{Q}_1 + \dot{Q}_2}{Nv} = \\ = \frac{\dot{U}_{e\Sigma}}{Nv} + \frac{\dot{Q}_\Sigma}{Nv}\,. \tag{22}$$

Here F_l; F_t and μ_l; μ_t; are the values of the force and coefficient of friction corresponding to the friction path l, and respectively their values per unit of time t.

Thus, the thermodynamic analysis of the process of plastic deformation and destruction of the volume of a solid body allows us to obtain generalized (binomial) relations for the work W_f and friction power $\dot{W}_f$, friction force F and friction coefficient μ, which corresponds to modern ideas about the dual nature of friction [8-10].

The balance equations (17)-(22) establish a deep mutual relationship of microscopic and generalized parameters $\Delta u_{e1}; \Delta u_{e2}; q_1; q_2$ of the friction process with macroscopic - external

characteristics of the friction process N and v. The proposed generalized friction equations take into account both the characteristics of the shaft material (counterbody) $\Delta u_{e1}; q_1$ - and the bearing material - $\Delta u_{e2}; q_2$.

In accordance with (13), the energy balance equations of the friction process (17) and (18) can be transformed to the form

$$W_f = \Delta U_{e1} + \Delta U_{e2} + \Delta U_{T1} + \Delta U_{T2} + \vec{Q}_1 + \vec{Q}_2 = = \Delta U_{e\Sigma} + \Delta U_{T\Sigma} + \vec{Q}_\Sigma; \quad (23)$$

$$\dot{W}_f = \dot{U}_{e1} + \dot{U}_{e2} + \dot{U}_{T1} + \dot{U}_{T2} + \dot{\vec{Q}}_1 + \dot{\vec{Q}}_2 = = \dot{U}_{e\Sigma} + \dot{U}_{T\Sigma} + \dot{\vec{Q}}_\Sigma. \quad (24)$$

Accordingly, for the friction force F and the coefficient of friction μ, equations (19) - (22) are also converted to the form

$$F_l = \frac{\Delta U_{e1} + \Delta U_{e2} + \Delta U_{T1} + \Delta U_{T2} + \vec{Q}_1 + \vec{Q}_2}{l} = = \frac{\Delta U_{e\Sigma}}{l} + \frac{\Delta U_{T\Sigma}}{l} + \frac{\Delta \vec{Q}_\Sigma}{l}; \quad (25)$$

$$F_t = \frac{\dot{U}_{e1} + \dot{U}_{e2} + \dot{U}_{T1} + \dot{U}_{T2} + \dot{\vec{Q}}_1 + \dot{\vec{Q}}_2}{v} = ; = \frac{\dot{U}_{e\Sigma}}{v} + \frac{\dot{U}_{T\Sigma}}{v} + \frac{\dot{\vec{Q}}_\Sigma}{v}. \quad (26)$$

$$\mu_l = \frac{\Delta U_{e1} + \Delta U_{e2} + \Delta U_{T1} + \Delta U_{T2} + \vec{Q}_1 + \vec{Q}_2}{Nl} = = \frac{\Delta U_{e\Sigma}}{Nl} + \frac{\Delta U_{T\Sigma}}{Nl} + \frac{\vec{Q}_\Sigma}{Nl}; \quad (27)$$

$$\mu_t = \frac{\dot{U}_{e1} + \dot{U}_{e2} + \dot{U}_{T1} + \dot{U}_{T2} + \dot{\vec{Q}}_1 + \dot{\vec{Q}}_2}{Nv} = = \frac{\dot{U}_{e\Sigma}}{Nv} + \frac{\dot{U}_{T\Sigma}}{Nv} + \frac{\dot{\vec{Q}}_\Sigma}{Nv}. \quad (28)$$

The relations between the components of the energy balance (23) - (28) of the friction process ΔU_{e1} and ΔU_{e2}, ΔU_{T1} and ΔU_{T2}, and also $\vec{Q}_1$ and $\vec{Q}_2$ vary widely and are determined by the physico-chemical properties of the materials composing the friction pair, their structure and the conditions of the friction process. Such a variety of particular relationships between the components of the energy balance of the friction process determines the entire variety of particular (boundary) manifestations of the friction and wear process.

Performing a thermodynamic analysis of the friction process, it is necessary to note one circumstance of a fundamental nature associated with the fundamental concept of tribology - the concept of the formation of an intersurface zone - the "third body" (Fig. 3) [1,8].

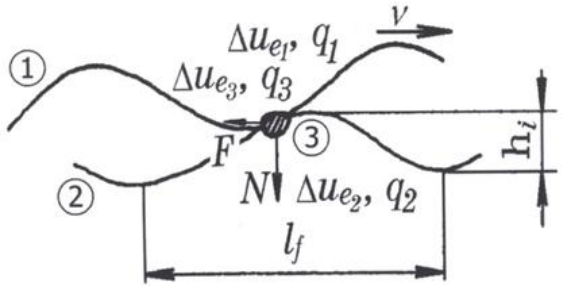


Fig. 3. Conditional scheme of metal contact during friction, taking into account the formation of the intersurface zone ("third body") [7].

In the most general case, the friction energy balance equations (17) and (18), taking into account this third component (Fig. 3), should be written as

$$W_f = \Delta U_{e1} + \Delta U_{e2} + \Delta U_{e3} + Q_1 + Q_2 + Q_3; \quad (29)$$

$$\dot{W}_f = \dot{U}_{e1} + \dot{U}_{e2} + \dot{U}_{e3} + \dot{Q}_1 + \dot{Q}_2 + \dot{Q}_3. \quad (30)$$

These relations take into account the components Δu_{e3} and q_3 characterizing the scheme of deformable contact, taking into account the emergence and existence of the "third body". For the special case of localization of friction in the volume of the "third body", equations (29) and (30) are transformed to the form

$$W_f = \Delta U_{e3} + \vec{Q}_3 = \Delta u_{e3} V_{f3} + \vec{q}_3 V_{f3}; \quad (31)$$

$$\dot{W}_f = \dot{U}_{e3} + \dot{\vec{Q}}_3 = \dot{u}_{e3} V_{f3} + \dot{\vec{q}}_3 V_{f3}. \quad (32)$$

This record of the equations of the energy balance under friction does not contradict the arguments given above, complements them and has a broader physical validity.

2.4 Structural-energy interpretation of the sliding friction coefficient

Now it is possible to propose a structural-energy interpretation of the sliding friction coefficient. For conditions characterized by low sliding speeds and an insignificant thermal friction effect ($Q \cong 0$), which corresponds to the experimental conditions of the first friction researchers, the friction energy balance equation (21) represents the coefficient of friction in a generalized form as

$$\mu = \mu_{adapt} = \frac{F}{N} = \frac{\Delta U_{ei}}{\mu^* N\, l} = \frac{V_f \Delta u_e}{\mu^* N\, l} = \frac{\Delta U_{ei}}{\Delta U_{e*}} \quad .(33)$$

Here $\mu^* = 1$; $Q \cong 0$.

This leads to the conclusion about the deep physical meaning of the friction coefficient of Leonardo da Vinci (G. Amontons).

On the one hand, it is a parameter that characterizes generically the resistance to relative displacement (movement) of surfaces, because it reflects the proportion of energy that is "destroyed" by friction in the form of stored potential energy in relation to the parameter $\mu^* N\, l$ - the work of external forces (energy of external relative motion).

On the other hand, it is a generalized characteristic of damage, because it (the coefficient of friction) is determined by the density of the latent potential energy Δu_e - an integral characteristic of the measure of the defect of the structure and is a generalized parameter of damage.

Here, the coefficient of friction unambiguously reflects the generalized structural perfection (imperfection) of the deformable contact volume, since the parameter $\Delta U_e = \Delta u_e \cdot V_f$ is determined by the density of potential energy of various kinds of structural defects and damages accumulated in the contact volumes V_f of the body. In accordance with the main conclusion of the thermodynamic theory of strength [2]), the coefficient of friction should be a generalized state parameter, since it is related to the structural state parameter - Δu_e.

Analyzing the ratio for the coefficient of friction (33), it can be seen that the friction force F is determined by the amount of accumulated latent, potential energy ΔU_e per unit of the friction path l, i.e.

$$F = \frac{\Delta U_{ei}}{l}. \quad (34)$$

Thus, the concept of friction resistance has a physical nature associated with the process of accumulation of potential energy ΔU_e by contact friction volumes and this is the energy "taken away" from the energy of external relative motion.

Based on the above-mentioned structural-energy interpretation of the friction process, it becomes possible to present the coefficient of friction in an expanded physical interpretation:

$$\mu = \frac{F}{N} = \frac{\Delta U_e}{N \cdot l} = \frac{\Delta U_e}{\mu^* N\, l} = \frac{V_f \Delta u_e}{\mu^* N\, l} =$$

$$= \frac{h\, A_r\, \Delta u_e}{\mu^* p_r\, A_r\, l} = \frac{h\, \Delta u_e}{\mu^* p_r\, l} = \frac{\Delta U_e}{F_* l} = \frac{F_i}{F_*} = \frac{\tau_i}{\tau_*}. \quad (35)$$

Therefore, the coefficient of friction is the ratio of the specific friction force τ_i to the maximum specific friction force τ_* corresponding to the maximum balance coefficient of friction $\mu^* = 1$. The balance coefficient of friction characterizes the condition of destruction (violation of the friction bond) of the friction volume V_f^*. This is a failure condition according to the theory of maximum tangential stresses - $\mu^* = 1 = tg45^0$. The procedure for determining the coefficient of friction, in fact, is a procedure for comparing it with the critical (balance) coefficient of friction, which characterizes the condition of violation of the frictional bond by volume V_f^*, as such, as a whole.

Moreover, it becomes possible to analyze friction taking into account the roughness parameter h (wear volume V_f) on a real friction contact. If the specific work of friction $p_r\, l$ is spent on the ultimate deformation hardening Δu_e^* of the

friction volume V_f^*, then the size of linear wear h on the real contact spot has a maximum, reference value equal to one. This unit is adequate to the balance, unit value of the coefficient of friction $\mu^* = 1{,}0$ at the point of extreme deformation hardening [11] of the critical volume of friction V_f^*.

3. AN ATTEMPT TO UNDERSTAND LEONARDO DA VINCI'S FORMULATIONS ABOUT THE COEFFICIENT OF RESISTANCE UNDER FRICTION

3.1 Why is the sliding ability of bodies different?

Before Leonardo da Vinci formulated the concept of the coefficient of friction (resistance (see introduction)) he writes the following: "The sliding ability of bodies is different, so the friction has a different magnitude. Bodies with a smoother surface have less friction."

Indeed, if we perform a friction experiment in a wide range of load or speed changes [1,8-10], then the coefficient of friction will change. What are the most common patterns of change in the coefficient of friction (resistance)?

A subsequent analyses of modern experimental data using equations (25)-(30) has shown that the experimental friction curves (Fig. 4-6) of type $\mu = \mu(N, v)$ are generalized friction curves that reflect the evolution (the change in the friction coefficient) of tribosystem.

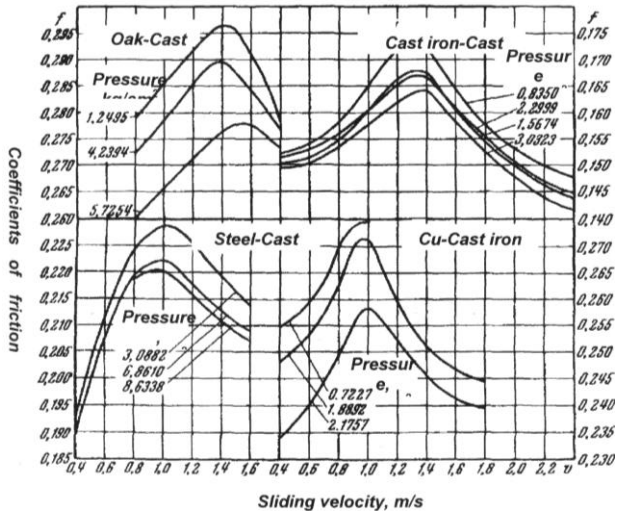


Fig. 4. P.Conti's experimental results [1].

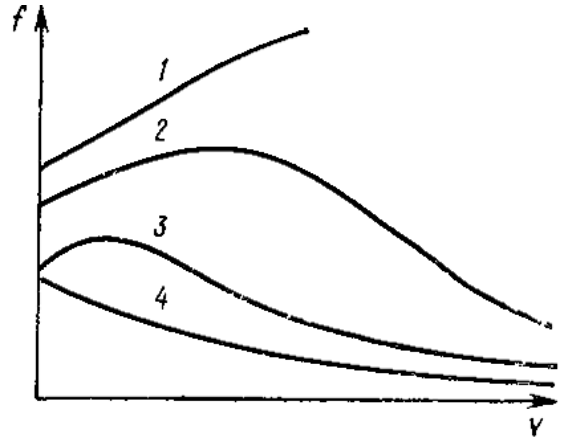


Fig. 5. Generalized friction experiments in Kragelsky's interpretation [8]: v - sliding velocity (load: 1-small; 2 and 3 – medium; 4 - considerable).

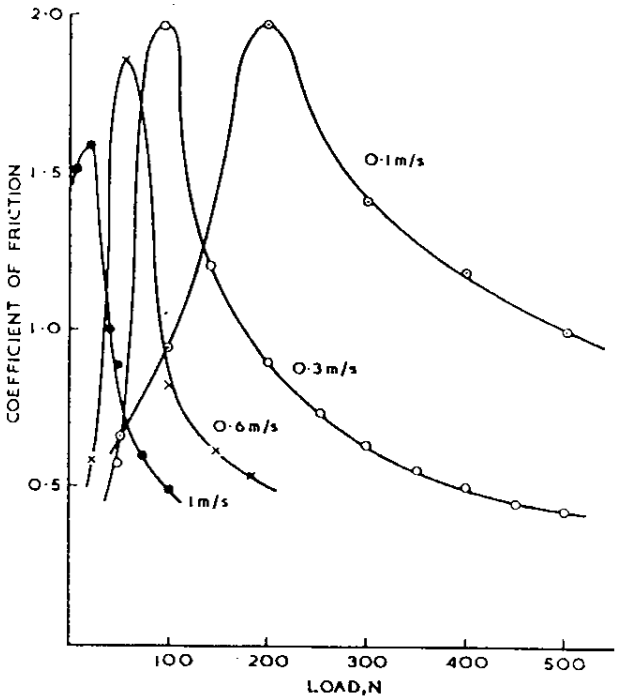


Fig. 6. Watanabe's [11] experimental results of Nylon 6 – Steel pair of friction.

We can consider here the representations [7,12] of the diagram of the evolution of rubbing surfaces (Fig.7). This diagram is constructed using the friction energy balance equations (7) – (33). There is an equilibrium at each point of the diagram and the balance principle is observed.

In the diagram, the evolution of the states and properties of the friction contact passes through a number of principle points 1-5. Between these points are the areas of the most characteristic transformations of the deformable contact.

The friction contact (Fig.4-7) is really evolving!

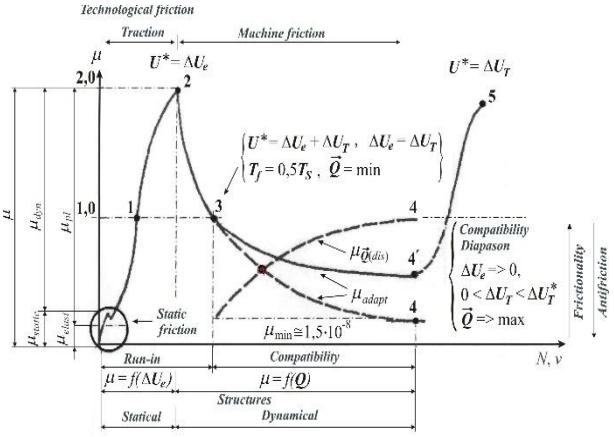


Figure 7. Structural-energy diagram of the evolution of rubbing surfaces [7,12]. Designation on the axes: N, v - load and speed; $\mu_{st}, \mu_{dyn}, \mu_{elast}, \mu_{plast}$ - static, dynamic, elastic, plastic coefficients of friction; T_f, T_S - flash point in the contact volume of friction in volume 3 and melting point.

So, you can see (Fig.7) the following points and areas: 0, area of static friction and strain hardening (dislocation mechanism); 1, the point of limiting strain hardening; 1–2, the area of excess energy pumping (vacancy mechanism); 2, the point of seizure and transition of external friction into internal (critical instability), a highly excited state; 2–3, the area of formation of dissipative structures (formation of a temperature flash in the friction volume); 3, the point of minimum compatibility (maximum frictionality); 1–3, area of self-organization; 3–4, area of compatibility; 4, point of wearlessness (abnormally low friction); 5, point of thermal seizure.

The differences in the individual states and properties of the friction contact are related to the peculiarities, for example, of the return of deformation energy and the degree of consistency of the processes of accumulation and release of energy over the time of the external disturbance. In some cases, the system (contact) only manages to accumulate the energy of external influence in the form of latent energy, and the release of energy occurs after the destruction of the system in wear particles. In other cases, the system (contact) manages to accumulate energy and release it during the action of an external disturbance.

Initially, when micro-processes of potential energy ΔU_e accumulation prevail, the coefficient of friction increases up to point 2. At this point, the process of forming a critical volume of friction – the volume of an elementary tribosystem - is completed. This is the smallest volume V_f that has accumulated energy of the maximum density u_{e*}. Further, when micro-processes of energy return begin to prevail, the coefficient of friction begins to decrease. In this region, beyond point 2, friction is localized in the volume of an elementary tribosystem, which is structurally evolving.

In this area, evolutionary restructuring has the features and peculiarities of physical and material structuring and is nothing more than dynamic recrystallization. On the other hand, the characteristic friction temperature at point 3, equal to $0{,}5T_S$, is on a par with the condition of recrystallization and the condition of structuring according to a chemical feature, i.e. it is a condition for chemical reactions to occur in the friction volume, due to interaction with the environmental substance at point 2, the activated friction surface (the effect of self-alloying).

In essence, the factor of the occurrence of a critical volume of friction is adequate to the concept of I.V.Kragelsky and V.S.Shchedrov [1,8] about the occurrence of equilibrium roughness after the process of running-in of friction surfaces.

In the area 3-4 in the diagram, we have a layer of dynamic dissipative structures on the intersurface of friction, which have adapted during the run-in process. Essentially, these are smooth surfaces (intersurface – «the third body»), i.e. actually polished surfaces.

As can be seen from the diagram (Fig.7), the area of points 3-4 is the area of compatibility, i.e. the area of joint work (deformation) of rubbing surfaces. This area exists after the surfaces are run-in and is essentially the area of machine friction (frictionality area up to the level $\mu_{adapt} = 0.5$ and anti-friction area from the level $\mu_{adapt} = 0.5$ and below).

3.2 Why should Leonardo da Vinci's coefficient of friction be equal to 0.25

It is quite obvious that Leonardo da Vinci, as a brilliant engineer, dealt with the problem of friction based on the interests of knowledge of machine friction. Apparently, his experimental studies of friction relate to the definition of the coefficient of friction as a parameter of resistance to the movement of rubbing surfaces.

If his formulation about the coefficient of friction, which should be equal to 0.25, refers to machine friction, then it seems appropriate to estimate the values of the coefficients of friction for the hinges of kinematic chains, for example, the simplest lever mechanisms (Fig.8).

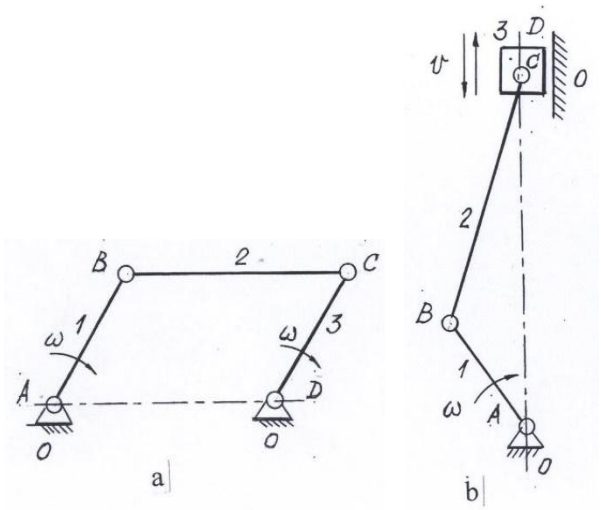


Fig. 8. Elementary machines – the simplest lever mechanisms: a) hinge parallelogram; b) crank-type and connecting rod mechanism.

These simplest mechanisms should be considered as the basis for all other mechanisms, i.e. as elementary machines.

At the SerbiaTrib19 conference, I made a report [13] on the topic of energy-entropy analysis of a machine (mechanism) as tribosupersystems. Actually, tribosystems of any machine are tribosubsystems. The machine itself (tribosupersystem) as a device having intrinsic motion of its parts, it is a transformer of both motion and energy.

In the article [13], the entire array of machines was divided into two principal classes – natural, i.e. ideal machines and machines designed by man as an analogue of the first.

The rule of a natural machine was proposed – the sum of the adaptive (Leonardo da Vinci) friction coefficients of the friction pairs of the machine in the compatibility area (Fig.7) is equal to one.

$$\sum_{1}^{n} \mu_{adapt\ i} = 1{,}0\,. \qquad (36)$$

Based on this rule, the conclusion follows: in any natural and constructed machine, regardless of their degree of complexity, the values of the friction coefficients of tribosystems (kinematic chain hinges) should be such that they add up to one.

The adaptive coefficient μ_{adapt} of compatible friction is an integral parameter of the state - resistance to movement and damage ability.

The dissipative coefficient of compatible friction $\mu_{dis_{\vec{Q}}}$ is the ability of dynamic dissipation, i.e. the ability to promote movement. The sum of the dissipative coefficients of friction of the machine, equal to an integer, is an indicator of the operability or some parameter of the reliability of the machine.

Table 2 below shows a number of values of the friction coefficients of natural machines.

Table 2. Possible set of natural tribosystems forming a machine (tribosupersystem).

$\mu_{adapt\ i}$	$\mu_{dis_{\vec{Q}}\ i}$	$n_{mach\ i} = \dfrac{1}{\mu_{adapt\ i}}$
0,5	0,5	2
0,25	0,75	4
0,2	0,8	5
0,1	0,9	10
0,05	0,95	20
0,04	0,96	25
0,025	0,975	40
0,02	0,980	50
0,01	0,990	100
0,005	0,995	200
0,004	0,996	250
0,0025	0.9975	400
0,002…	0,998…	500…

And so on.

As follows from Table 2, in the simplest machine – a lever mechanism with four links $n_{mach\ i}$, the friction coefficients of the hinges are 0.25.

I believe that this fact may well answer the question why, in Leonardo da Vinci's formulation, for the coefficient of friction of any pair of friction with polished surfaces, the coefficient of friction should be equal to 0.25.

It would be more correct here to talk about a coefficient of friction of no more than 0.25. Less is possible and this is the rule of forming a machine in order to increase its efficiency.

As a result, it can be assumed that Leonardo da Vinci, in the formulation about the coefficient of friction equal to 0.25, apparently meant precisely friction in an elementary machine (the simplest mechanism). As can be seen from the diagram of rubbing surfaces in Fig. 7, machine friction with a coefficient of friction of 0.25 is realized in the compatibility zone, where surfaces precisely have the property of smoothness and polish - dynamic dissipative friction structures.

In this sense, Leonardo da Vinci's logic is really mysterious. If we managed to solve this riddle, then there is a feeling of delight in the talent of this great man!

4. CONCLUSION

1. The friction experiment accumulated by modern tribology confirms Leonardo da Vinci's opinion that every frictional body has a different ability to slip. The structural-energy analysis of the friction process substantiates the general patterns of changes in sliding resistance by the evolutionary nature of the friction contact.
2. Thermodynamic analysis of the friction process and structural-energy interpretation of the coefficient of friction shows that the resistance to the movement of rubbing surfaces is determined by the amount of accumulated internal (potential) energy by contact volumes.
3. This potential (static) energy of various kinds of elementary defects and damages to the structure of friction volumes, in the process of friction itself as a transformational process, is "taken away" from the energy of the external relative motion of the friction surfaces.
4. The friction force and the coefficient of friction as a measure of resistance to the movement of rubbing surfaces are determined by the energy parameter of the process itself – the accumulated potential energy of elementary structural defects and damages of deformed contact friction volumes.
5. If the logic of our study of machine friction in the simplest mechanisms, discussed above, is correct, then we can only admire Leonardo da Vinci's foresight about the magnitude of the coefficient of friction in the simplest mechanisms (elementary machines).
6. How he could have foreseen this is really an amazing mystery of genius intelligence!

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